

Quartic Equation and proportion of physical reality

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ABSTRACT

Considering a quartic equation as polynomial of the 4th order with non-zero coefficients a, b, c, d & e can be considered as a sum of partial equations. In order to express coefficients of these equations with the help of functional dependence on a given parameter K , solution of individual partial equations can be found, using the numerical values for the sizes of regular 4-dimesinal polyhedron surfaces will then determine their volume ratios. These ratios will also be matching the mass ratios of fundamental particles of the physical realm as for example is the mass ratio of a proton and an electron or the mass ratio of Higgs boson and an electron including the Planck and the electron mass ratio. These mass proportionality in physical realm in terms of elementary particles then enables the solution of partial equations to determine black hole the final “gravity” after the collapse of two mass objects. By these proportions the most massive objects are determined whereas from the initial mass of two objects the mass of gravitationally collapsed object can also be established.

From those equations a new elementary particle representing the dark matter could be found with likelihood of finding other particles that could explain the essence of dark energy. On the bases of gravitational objects represented by a black hole, the property of those initial objects that exclude collapse into singularity could be determined.

Keywords: astroparticle, black hole physic

1 INTRODUCTION

In this paper partial equations and their sums are defined and their solutions found for the defined comon condition. The found roots of quadratic and cubic equations are then substituted as sides of regular multisided bodies¹ in the 4 dimensional space. Resulting volume ratios of the selected multisided bodies are then compared with rations of non-moving masses of certain elementary particles Particle Data Group (Tanabashi et al. 2018) that are part of the standard model of microworld. These volume rations show agreement with rations of non-moving masses that could be understood as proportions of the physical reality. The quadratic equation of its own defined as the „gravitational“ enables us to find constants that are characterising the resulting mass of black holes. Similarly as when using the method of numerical solutions (Abbott, et. al. 2018). By the same way the proportions of the physical microworld can be described as well as proportions of mass objects like black holes.

2 QUADRATIC EQUATION WITH VARING COEFFICIENTS

Using the general form of a quadratic equation (Brož & Roskovec, 1987) in the form

$$ax^4 + bx^3 + cx^2 + dx + e = 0 \quad (1)$$

In order to find a solution of the above defined partial equations we substitute $x=R$. This being in direct reference to the gravitational radius of massive objects, black holes in particular, where $R=r/r_g$. Where $r_g = Gm/c^2$ is well known gravitational radius of an object with mass m , where G is gravitational constant and c being the speed of light.

After substituting coefficiets we get

$$a=K^2(K^2-4), b=K^2(18-6K^2), c=13K^4-26K^2+1, d=12K^2(1-K^2) \\ a e=4K^4,$$

where parameter K is the coefficient that represent the force size between objects. It significance will be shown in the following section of this paper. The general equation (1) can then be rewritten in the form

$$K^2(K^2 - 4)R^4 + 6K^2(3 - K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(1 - K^2)R + 4K^4 = 0 \quad (2)$$

Using the above substitutions we have defined the equation of „vacuum“. The use of the inverted comers is chosen deliberately, and similarly for other definitions so it can be used as a guide when comapring other numerical values with similar numerical values for physical reality. These values also include mass ratios of selected elementary particles. Using even powers only for parameter K , it is thus sufficient using only positive K values.

¹https://cs.wikipedia.org/wiki/%C4%8Cty%C5%99roz m%C4%9Brn%C3%A1_plat%C3%B3nsk%C3%A1_t%C4%9Blesa

3 LIMITED CHARACTERISTICS OF THE EQUATION FOR „VACUUM“

Taking $R=0$ as a limiting factor, when the parametner $K=0$ is determined by the equation (2). In the physical sence it denotes zero interaction at the distance $r=0$. In order to find the solution for limiting factor $R \rightarrow \infty$, thus $R \neq 0$, we can then rewrite equation (2) multiplying by $1/R^4$ and getastroparticle, bleck hole physics

$$K^2(K^2 - 4) + 6K^2(3 - K^2)\frac{1}{R} + (13K^4 - 26K^2 + 1)\frac{1}{R^2} + 12K^2(1 - K^2)\frac{1}{R^3} + 4K^4\frac{1}{R^4} = 0.$$

Thus formed equation for limiting factor $R \rightarrow \infty$ determines $K=2$. We can when parameter $K=2$ call it the „annihilation“ parameter and use it when looking for solution of other partial equations. It also becomes meaningfull when $K=2$ for limiting factor $R \rightarrow \infty$.

After finding the solution of equation (2) for limiting factor $K \rightarrow \infty$, thus $K \neq 0$. This equation when multiplying it by $1/K^4$ becomes

$$\left(1 - \frac{4}{K^2}\right)R^4 + \left(\frac{18}{K^2} - 6\right)R^3 + \left(13 - \frac{26}{K^2} + \frac{1}{K^4}\right)R^2 + \left(\frac{12}{K^2} - 12\right)R + 4 = 0.$$

For $K \rightarrow \infty$ the above equation can be simplified to

$$R^4 - 6R^3 + 13R^2 - 12R + 4 = 0.$$

This equation becomes meaningfull when $R_1=2$ and also when $R_2=1$, thus for $r_1=2r_g$ and also when $r_2=r_g$. These r values play important part in Newton's understading of the gravitation as well A Einstein's general theory of relativity where it determines both Schwazschilder's and gravitational diameter.

Special solution of the „vacuum“ equation (2) for the „annihilation“ parameter $K=2$ will be shown after difining partial quations and their solutions. Further defined equations when added up the equation (2) is optained.

4 DEFINITION OF THE PARTIAL „FERMION“ EQUATION AND ITS SOLUTION.

We assume that „vacuum“ equation (2) is given be summation of three partial equations. The first being named the „fermion“ equation in the form:

$$(13K^4 - 26K^2 + 1)R^2 - 12K^2(K^2 - 1)R + K^4 = 0 \quad (3)$$

Solving this quadratic equation for the „annihilation“ parameter $K=2$ and substituting it into the equation (3) we obtain

$$105R^2 - 144R + 16 = 0, \quad (4)$$

whose two roots we will label $R_{F1}=1,249472327$ and $R_{F2}=0,1219562443$. These roots of equation (4) have the following attribute

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$\frac{1}{6} \frac{R_{F1}^4}{R_{F2}^4} = 1836,281044$. This ratio is with a small deviation in agreement with the mass ratio of a proton and an electron 1836,15267389 (Tanabashi et al. 2018)

From the above R_{F2} is regarded as a side length of tesseract in 4-dimensional space whose volume is defined by fourth power of the side length. One sixth of the fourth power of the side R_{F1} is the volume of 16cell of the same space (volume), this being dual to tesseract.

5 DEFINITION OF THE PARTIAL „BOSON“ EQUATION AND ITS SOLUTION

The „boson“ equation is formed by the summation of the „fermion and „gravitation“ equations for „vacuum“ and then becomes the cubic equation

$$K^2(18 - L)R^3 - (13K^4 - 26K^2 + 1)R^2 - 12K^2(K^2 - 1)R - K^4 = 0. \quad (5)$$

Significance of the parameter L will become clear when the third equation is solved as shown below. For $L=18$ the equation (5) changes into the quadratic form.

$$(13K^4 - 26K^2 + 1)R^2 + 12K^2(K^2 - 1)R + K^4 = 0 \quad (6)$$

whose two roots when $K=2$ are in agreement with the roots of the equation (4) except for the change of sign.

The ratio $\frac{1}{6} \frac{R_{B1}^4}{R_{F2}^4} = 1836,281044$ has the same value as is the case in the equation (3).

Now let us solve the equation (5) for different parameters L . First for $L=9,767$ the value of which will be used later in the case that support the choice of this value.

After substituting $K=2$ in the equation (5) the equation can be re-written in the standard form

$$R^3 - \frac{105}{4(18-L)}R^2 - \frac{144}{4(18-L)}R - \frac{16}{4(18-L)} = 0. \quad (7)$$

After substituting for L the value found by solving the cubic equation (7) we obtain three factual roots.

$R_{B1}=4,245334\dots$, $R_{B2}=-0,934478\dots$ a $R_{B3}=-1,2246\dots$. The ratio of volumes $\frac{1}{6} \frac{R_{B1}^4}{R_{F2}^4} = 2,44726 \cdot 10^5$.

Using the above figure and multiplying it by the mass of an electron $510,9989461 \text{ keV}c^{-2}$ we obtain the mass of Higgs boson $125,05475 \text{ GeV}c^{-2}$ (Tanabashi et al. 2018). The volume ratio of polyhedrons with sides R_{B1} and R_{F2} agree with the ration of the resting mass of Higgs boson and that of an electron. Special solution of the equation (5) for value close to 18 where parameter $L=17,999707558$ we obtain 3 factual roots i.e. $R_{B1}=89762,75999\dots$, $R_{B2}=-1,249453\dots$ and $R_{B3}=-0,121956284707267$. These figures agree with volume ratios $\frac{1}{6} \frac{R_{B1}^4}{R_{F2}^4} = 4,8912219891 \cdot 10^{22}$. Numerical ration of these volumes agrees with numerical ration of the Planck and

Quartic Equation and proportion of physical reality electron masses, that is $m_p = 2,499409289 \cdot 10^{19} \text{ GeV}c^{-2}$. The ratio $\frac{1}{6} \frac{R_{B2}^4}{R_{F2}^4} = 1836,1676$.

The ratio of these volumes is numerically close to the mass ratio of a proton and an electron, where $m_p/m_e = 1836,15267389$ (Tanabashi et al. 2018) while $m_p = 938,2720813 \text{ MeV}c^{-2}$. While $R_{F2}+R_{B3} = -4,04 \cdot 10^{-8}$, the solution of equation (5) for parameter $L=18,00619440927$, we get three factual roots $R_{B1} = -0,1219558644\dots$, $R_{B2} = -4236,32038\dots$ and $R_{B3} = -1,2498880831434211$. This R_{B2} figure agrees with the volume ratio $\frac{1}{6} \frac{R_{B2}^4}{R_{F2}^4} = 2,4265 \cdot 10^{17}$. Providing we multiply this ratio with the mass of an electron we get $1,23996 \cdot 10^{14} \text{ GeV}c^{-2}$. Value of R_{B3} agrees with the volume ratio $\frac{1}{6} \frac{R_{B3}^4}{R_{F2}^4} = 1838,683562$.

This ratio of volumes with a small deviation is getting close to the numerical value of a neutron and an electron masses where $m_n/m_e = 1838,683661$ and $m_n = 939,565413 \text{ MeV}c^{-2}$ (Tanabashi et al. 2018). Similarly as in the previous case we find that $R_{F2}+R_{B1} = 3,799 \cdot 10^{-7}$.

6 DEFINITION OF THE PARTIAL „GRAVITATIONAL“ EQUATION AND ITS SOLUTION

Following the third equation obtained by summation of equations (3) and (5) we obtain equation (2) in the form

$$K^2(K^2 - 4)R^4 + K^2(L - 6K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(K^2 - 1)R + 4K^4 = 0 \quad (8)$$

The equation (8) can be re-written for $K=2$ in the form

$$R^3 + \frac{105}{4(L-24)}R^2 + \frac{144}{4(L-24)}R + \frac{64}{4(L-24)} = 0 \quad (9)$$

The equation (9) can then be use to solve for any arbitrary value of L . However, when comparing volumes of regular polyhedrons in 4 diemsional space and when comparing it with masses of particles in the microworld we can solve this equation for two chosen values of L . First we find the solution when $L=24$. For this case the equation (9) changes into the quadratic equation.

$$105R^2 + 144R + 64 = 0 \quad (10)$$

The equation (10) has two complex roots. This solution is the boundary limit between solution equation (9) for $L > 24$ and solution for $0 < L < 24$. In this interval we are interested in the solution of equation (9) for $L=18,006195695$ followed by $R_{G1} = 5,54868083091149$ and further for $\frac{1}{6} \frac{R_{G1}^4}{R_{F2}^4} = 7,14153971 \cdot 10^5$. These volume ratios with a small deviation is close to a quadruple mass of intermediate boson Z^0 to the mass of an electron, where $4m_z/m_e = 7,13798 \cdot 10^5$ where $m_z = 91,1876 \text{ GeV}c^{-2}$.

The equation (9) has two special solution. When $L=17,875$, we can obtain one root $R_{G1}=2$ and two roots

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$R_{G2}=1,142857097002739$ and $R_{G3}=1,1428578871115$. For the root R_{G1} we get $\frac{R_{G1}^4}{R_{F2}^4}=72327,56$. This ratio corresponds to the mass of $36,9593 \text{ GeV}/c^2$.

In the case of roots R_{G2} a R_{G3} the volume ratio $\frac{R_{G2}^4}{R_{F2}^4}=7711,725$. In this case the equivalent mass being multiple of an electron mass that equates to $3,94 \text{ GeV}/c^2$.

Bearing in mind the numerical value of roots R_{G2} a R_{G3} mutually differ numerically up to by the order of seven decimal places and in the same order will differ the mass, here rounded to $3,94 \text{ GeV}/c^2$.

Second special solution of the equation (9) for $L=17,75$ the first root being $R_{G1}=1,600000003441276$, the second root $R_{G2}=1$ and the third root $R_{G3}=1,599999996558724$. In this case the volume ratio being $\frac{R_{G2}^4}{R_{F2}^4}=4520,47$ with the equivalent mass of $0,385 \text{ GeV}/c^2$. For both roots R_{G1} a R_{G3} volume ratio is $\frac{R_{G1}^4}{R_{F2}^4}=183195,12$. By multipling the mass of an electron with the given figure we get mass of $93,61 \text{ GeV}/c^2$, that can be comapred with the mass of the intermediate mezonu $Z^0=91,1876 \text{ GeV}/c^2$.

7 THE EQUATION OF „VAKUUM“

The „vakuum“ equation (2) is obtain by adding up three previous equations „fermion“ (3), „boson“ (5) and „gravitational“ (8). In the introduction the range of validity is mentioned for $0 \leq R < \infty$ and parametr $0 \leq K < \infty$. At the same time the use of parametr $K=2$ is justified. Solving equation (2) and similarly as equations (3), (5) and (6) for $K=2$. By substituting into the equation (2) the „vakuum“ equation can be written in the stadard form

$$R^3 - \frac{35}{8}R^2 + 6R - \frac{8}{3} = 0 \quad (11)$$

The equation (11) has one real and two complex roots. The real root $R_{V1}=2,19496887746\dots$ is valid for $\frac{1}{6} \frac{R_{V1}^4}{R_{F2}^4}=17488,2378$ or provided we use the volume ration of tesseract with given sides, then $\frac{R_{V1}^4}{R_{F2}^4}=104929,4268$. From physical point of view the vakuum as such is interpreted as a range of virtual particles that can appear in some actual proceses. Gravitina and graviton could be such virtual particles that constitute the gravitational field. From the above mentioned numerical values we could speculate about gravitin with lower mass $m_{gd}=8,93647 \text{ GeV}/c^2$ because $m_{gd}/m_e=17488,2378$ and also gravitin with upper mass $m_{gu}=53,6188 \text{ GeV}/c^2$ because $m_{gu}/m_e=104929,4268$. This speculation is based on similar arrangement of the fermion couple thus an electron and a proton carrying elementary electrical charge. In case of gravitin their masses are elementary entity of the „gravitational charge“.

8 SUMATION OF THE „FERMIONS“ EQUATION (3) AND THE „BOSONS“ EQUATION (5)

Adding up equation (3) and (5) we obtain the equation

$$K^2(18 - L)R^3 - 24K^2(K^2 - 1)R = 0 \quad (12)$$

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Its solution has three real roots $R_{FB1}=0$ for any K . Thus ration $\frac{R_{FB1}^4}{R_{F2}^4}=0$.

If we consider that the stationary mass of a foton is zero than its ratio corresponds to $m_f/m_e=0$.

We can find two factual roots for $K=2$ as $R_{FB2,3}=\pm\sqrt{72}/(18-L)$. Chosing parametr $L=9,369258288$ will define the ratio $\frac{R_{FB2,3}^4}{R_{F2}^4}=314595,5584$.

This volume ratio of tesserakts is equal to the mass ratio $2m_{W\pm}/m_e=314595,5584$ from which $m_{W\pm}=80,379 \text{ GeV}/c^2$. The mass ratio of an electron and to double mass of intermediate boson $W\pm$.

9 SUMATION OF THE „FERMION“ EQUATION (3) AND THE „GRAVITATION“ EQUATION (8)

Adding up equations (3) and (8) we get

$$K^2(K^2 - 4)R^4 + K^2(L - 6K^2)R^3 + 2(13K^4 - 26K^2 + 1)R^2 + 5K^4 = 0 \quad (13)$$

Considering $K=2$ after some manipulation we obtain standard cubic equation without a linear part.

$$R^3 + \frac{210}{4(L-24)}R^2 + \frac{80}{4(L-24)} = 0 \quad (14)$$

The solution of equation (14) has one real root and two complex roots when $L=0$.

The real root $R_{FG1}=2,339725\dots$ and its corresponding volume ratios $\frac{1}{6} \frac{R_{FG1}^4}{R_{F2}^4}=22579,67$. To this numerical value of ratios corresponds the particle mass of $11,5375 \text{ GeV}/c^2$ in ratio to the mass of an electron. For $L=18$ the equation (14) has one real root $R_{FG1}=8,791115\dots$ and corresponding volume ratios $\frac{1}{6} \frac{R_{FG1}^4}{R_{F2}^4}=4,504 \cdot 10^6$. To this numerical value of ratios corresponds the mass particle $2,3 \text{ TeV}/c^2$ in ratio to the electron mass.

The special case of the equation (14) solution, which we multiply by a factor $4(L-24)$, for $L=24$, the imaginary root $R_{FG1}=80/210i$. The forth power, thus the volume of a regular polyhedron in four dimensional space is positive and represent the regular volume. This volume when multiplied by the numerical coeficient of a corresponding regular polyhedron in four dimensional space can be used to determine the volume ratio $R_{FG1}^4=0,02106118335$. Because of the existance of six regular polyhedrons in four dimensional space we obtain six volume ratios that correspond to six mass particles. The individual volume ratios are as follows:

5cell- tesseract $\frac{\sqrt{5} R_{FG1}^4}{96 R_{F2}^4}=2,21758\dots$ and corresponds to the mass of $1,133 \text{ MeV}/c^2$,

16cell-tesseract $\frac{1}{6} \frac{R_{FG1}^4}{R_{F2}^4}=95,2065\dots$ and corresponds to the mass of $8,108 \text{ MeV}/c^2$,

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8cell-tesseract $\frac{R_{FG1}^4}{R_{F2}^4}=15,86775\dots$ and corresponds to the mass of $48,65 \text{ MeVc}^{-2}$,

24cell-tesseract $2\frac{R_{FG1}^4}{R_{F2}^4}=190,413\dots$ and corresponds to the mass of $97,3 \text{ MeVc}^{-2}$,

120cell-tesseract $\sqrt{(2207 + 987\sqrt{5})\frac{1125}{8}\frac{R_{FG1}^4}{R_{F2}^4}}=75009,106$
and corresponds to the mass of $38,3 \text{ GeVc}^{-2}$,

600cell-tesseract $\frac{25(2+\sqrt{5})}{4}\frac{R_{FG1}^4}{R_{F2}^4}=2520,63\dots$ and corresponds to the mass of $1,288 \text{ GeVc}^{-2}$.

Considering the last three most massive volume ratio from above we obtain average $(0,0973+38,3+1,288)/3=13,228 \text{ GeVc}^{-2}$. Providing we look at the value representing the dark energy at $\approx 68\%$ then when considering $k \approx 5\%$ of barion mass then the mass of barion particle is $5 \cdot 13,228/68=0,977 \text{ GeVc}^{-2}$ which is almost the same as the mass of proton $0,936 \text{ GeVc}^{-2}$. When considering the imaginary type $R_{FG1}=80/210i$ it can be speculated that the dark energy is in essence as an effect of this mixture of these „virtual“ masses as background of physical vacuum.

10 SUMATION OF THE „BOSON“ EQUATION (5) AND THE „GRAVITATION“ EQUATION (8)

Adding up equation (5) and (8) we get

$$K^2(K^2 - 4)R^4 + K^2(18 - 6K^2)R^3 + 3K^4 = 0 \quad (15)$$

Considering $K=2$ and after some manipulation we obtain equation.

$$-24R^3 + 48 = 0 \quad (16)$$

The solution of the equation (14) shows one real root $R_{BG}=3\sqrt{2}$.

Volume ratio of the tesseract with the site R_{F2} to the volume of tesseract with the site R_{BG} has a numerical value $\frac{R_{BG}^4}{R_{F2}^4}=11387$. If we multiply this value with the mass of an electron we then obtain mass of $5,819 \text{ GeVc}^{-2}$, which corresponds to six times the mass of a proton thus that equates approximately to six times of the barion mass when the composition of the universe is considered. Considering the above we can assume that a particle with this mass is the particle of the dark matter we search for. This would suppose that these particles have the same numerical density as the barion in the composition of the universe.

11 SOLUTION OF THE „GRAVITATIONAL“ EQUATION FOR BLACK HOLES

Equation (8) in the form

$$K^2(K^2 - 4)R^4 + K^2(L - 6K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(K^2 - 1)R + 4K^4 = 0$$

We can first solve for $R_1=2$ and $L=9,767$, that we used to solve the „boson“ equation (5) to determine the volume ratio that is

Quartic Equation and proportion of physical reality equal to the mass ration of an electron and the mass of a Higgs boson. This solution enables us to determine the mass of a black hole.

For given values of R_1 and L we find corresponding value of $K_1=0,18885409673659085$. The same value of parameter K_1 also corresponds in this equation for $R_3=0,012362917264635$. For given R_3 solution of the equation also corresponds for $K_2=0,0321331428386$. Values of K_1 and K_2 can be used to calculate directly the mass of the black hole that is created by the collapse of two objects with mass m_1 a m_2 , where $m_1 \geq m_2$. The mass of the black hole we can then determine by the following equation

$$M_S = m_1 + C_S m_2 \quad (15)$$

where

$$C_S = 1 - \frac{(K_1+K_2)}{2} \sqrt{1 + 2\sin\omega\cos\omega} \quad (16)$$

and ω is combined angle of the average of the two parameters K_1 a K_2 . With the help of the resultant mass of the black hole M_S , where index S denotes the mass acquired by combination of two states, the equivalent mass of the gravitational waves we can be determined from

$$M_{gws} = m_1 + m_2 - M_S. \quad (17)$$

Values of M_S , C_S and M_{gws} defined by equations (15), (16) a (17) are shown in the Table 1 together with entry parameters of masses m_1 , m_2 and the parameter ω . The unit of mass is the mass of the Sun as quoted in the original collaborative work of LIGO-Virgo ([Abbott, et. al. Nov 2018](#); [Abbott, et. al. Jan 2020](#); [Abbott, et. al. Apr 2020](#)).

Figures for the mass of black holes M_{gtr} were taken from the above publications together with emitted gravitational waves M_{gwgtr} for masses of collapsing objects m_1 and m_2 . Ratios M_S/M_{gtr} and M_{gws}/M_{gwgtr} show a degree of agreement with calculation presented in this paper.

The second solution of the equation when $R_2=1$ and $L=9,767$. For the above values of R_2 and L the value of $K_3=0,1782579959600742$ was found. The resultant K_3 also agrees with this equation when $R_4=0,01100197289968$. The solution of this equation for resulting R_4 also agrees for $K_4=0,0303569329137$. Combination of these parameters is not discussed in this paper since the above applies only for distances smaller than $2r_g$.

They are nonsingular inner states of a black hole, that could be described in terms of elementary particles in connection with densities of given volumes multiplied by $0,011r_g$.

Value of $R_4=0,01100197289968$ in connection with volume ratio is notable. Since $\frac{\sqrt{5}}{96}\frac{R_4^4}{R_{F2}^4}=1,54269 \cdot 10^{-6}$ (the ratio of 5 regular four dimensional 5cell and regular four dimensional 16cell), then multiplying mass of an electron by this number defines the mass of $0,7883 \text{ eVc}^{-2}$. This mass is in the area of predicted neutrinos masses.

Bearing in mind the existence of independent solution of equation (8) for $R_1=2$ a $R_2=1$ it is possible to choose when trying to determine the mass of a black hole extended

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mixture of states represented by parameters K_3 a K_4 . It is however not addressed in this paper.

12 SUMMARY SOLUTION OF DEFINED EQUATIONS FOR $K=2$

In order to facilitate the overview of obtained solutions of individual equations and compared volumes of regular polygons in four dimensional space the results are arranged and listed in Table 2. The first column of the table refer to equations that are dependant on the parameter L . The second column in the table are copied values of volume rations of regular polygons in four dimensional space. The sides of these polyons have been obtained by solving the the above given equations that were simplified by substitution for $K=2$ to cubic eqations and a quadratic equation in case of varying R . The values in the third column were obtained by multiplying resting mass of an electron where $m_e=0,5109989461 \text{ MeVc}^{-2}$. The forth column is used to designate the calculated values using the conventional name/title for particles used in the standard model.

13 CONCLUSION

Standard solution of partial quations when added up determine quartic equation with changing coefficients introduced in this paper have suprising congruence with proportions of physical reality. This was achieved by substituting $R=r/r_g$ and this mathematical solution enable us to compare it with proportions of microworld in sense of finding congruence of volume rations of regular polygons with mass rations of selcted elementary particles with the mass of an electron. Chosing an electron was as a result of being the lightest of the known elementary particles. This was particularly due to the mass ration of an electron and a proton, a neutron mass and an electron, the mass of Higgs boson to the mass of an electron and the mass of an electron and particles of intermedial bosons W^\pm , Z . The same way it is possible to determine the Plancks' mass, which is the combination of the universall constant c (speed of light), h (Plancks constant) and G (gravitational constant). The wider group of equation solutions and the interpretation of volume rations includes the predication of probable mass of particle identy of dark energy and probable a virtual mass particle identy of the dark energy and the estimate of neutrino mass.

The „boson“ equation enable us to determine the the mass of a proton just by knowing the universal constant c , h , G and by their combination to determine the Plancks mass. The „ fermion“ equation then based on knowing the mass of a proton determines the mass of an electron.

The congruence of volume ratio with masses of elementary particles is not incidental, since the special solution of partial equation for graviation enable us to form mathamatical formula to calculate the mass of a black hole after the colapse of two mass objects. In such a case it can be considered as being a proportion of the macroworld. The calculated masses of black holes are in general congruent with masses that are quoted in documents LIGO-Virgo ([Abbott, et. al. Nov 2018](#); [Abbott, et. al. Jan 2020](#); a [Abbott, et. al. Apr 2020](#)).

Simplified names of individually defined eqations and their sumation are used in Table 2, which also shows the defined rations of regular polygons and their multipliers of the mass

Quartic Equation and proportion of physical reality of electron that is $0,5109989461 \pm 0,0000000031 \text{ MeVc}^{-2}$ (Particle Data Group ([Tanabashi et al. 2018](#)).

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DATA AVAILABILITY

The datasets were derived from sources in the public domain:

[dataset] *[B. P. Abbott](#), et. al., Nov 2018, arXiv1811.12907, 12 DOI: 10.1103 / PhysRevX.9.031040 <https://arxiv.org/abs/1811.12907>

[dataset] *[B. P. Abbott](#), et. al., Jan 2020, arXiv2001.01761, 9 DOI: [10.3847 / 2041-8213 / ab75f5 https://arxiv.org/abs/2001.01761](https://arxiv.org/abs/2001.01761)

[dataset] *[B. P. Abbott](#), et. al., Apr 2020, arXiv2004.08342, 6 <https://arxiv.org/abs/2004.08342>

[dataset] *[M. Tanabashi et al.](#), 2018 (Particle Data Group) DOI: 10.1103/ [PhysRevD.98.030001](https://arxiv.org/abs/2004.08342)

[dataset] *[P.A.R. Ade et. al.](#), 2013, arXiv1303.5062 Planck 2013 results. XVI. Cosmological parameters, 11 DOI: [10.1051 / 0004-6361 / 201321529https://arxiv.org/abs/1303.5062](https://arxiv.org/abs/1303.5062)

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[Abbott BP](#), et. al., Nov 2018, [arXiv1811.12907](https://arxiv.org/abs/1811.12907), 12

[Abbott BP](#), et. al., Jan 2020, [arXiv2001.01761](https://arxiv.org/abs/2001.01761), 9

[Abbott BP](#), et. al., Apr 2020, [arXiv2004.08342](https://arxiv.org/abs/2004.08342), 6

Brož J. & Roskovec V. 1987, [Přehled užitě matematiky](#)

Tanabashi M et. al. 2018 Particle Data Group, [Phys. Rev. D 98, 030001, 127](#)

Ade PAR, et. al., 2013, [arXiv1303.5062](https://arxiv.org/abs/1303.5062) Planck 2013 results. XVI. Cosmological parameters, 11

Object	m_1	m_2	ω	»	C_s	M_s	M_{gws}	M_{gtr}	M_{gwgtr}	M_s/M_{gtr}	M_{gws}/M_{gwgtr}
GW150914	35,6	30,6	94,600	»	0,8987	63,1009	3,0991	63,1	3,1	1,000	1,000
GW151012	23,3	13,6	90,100	»	0,8897	35,3999	1,5001	35,7	1,5	0,992	1,000
GW151226	13,7	7,7	78,800	»	0,8701	20,4001	0,9999	20,5	1	0,995	1,000
GW170104	31	20,1	90,550	»	0,8906	48,9005	2,1995	49,1	2,2	0,996	1,000
GW170608	10,9	7,6	85,700	»	0,8815	17,5996	0,9004	17,8	0,9	0,989	1,000
GW170729	50,6	34,3	71,400	»	0,8600	80,0992	4,8008	80,3	4,8	0,997	1,000
GW170809	35,2	23,8	88,450	»	0,8866	56,3001	2,6999	56,4	2,7	0,998	1,000
GW170814	30,7	25,3	91,900	»	0,8932	53,2987	2,7013	53,4	2,7	0,998	1,000
GW170817	1,46	1,31	123,730	»	0,9695	2,7300	0,0400	2,73	0,04	1,000	1,000
GW170818	35,5	26,8	94,850	»	0,8992	59,5997	2,7003	59,7	2,7	0,998	1,000
GW170823	39,6	29,4	89,100	»	0,8878	65,7009	3,2991	65,6	3,3	1,002	1,000
GW190425*	2,39	1,195	53,000	»	0,8453	3,4001	0,1849	3,4	0,185	1,000	1,000
GW190425*	1,88	1,595	90,400	»	0,8903	3,3000	0,1750	3,3	0,175	1,000	1,000
GW190412**	31,7	8	73,350	»	0,8625	38,5998	1,1002	38,6	1,1	1,000	1,000
GW190412**	27,5	9	76,400	»	0,8666	35,2996	1,2004	35,3	1,2	1,000	1,000
GW190412**	29,7	8,4	78,100	»	0,8691	37,0004	1,0996	37	1,1	1,000	1,000

*) Variation of entry values m_1 & m_2 for GW190425.

**) Variation of entry values m_1 & m_2 for GW190412.

Table 2

Equation for $K=2$	Values of volume rations	Multiplier of electron mass	Conventional particle designation
"vacuum" (2)	17488,2378	8,93647 GeVc ⁻²	? (gravitino down)
"vacuum" (2)	104929,4268	53,6188 GeVc ⁻²	? (gravitino up)
"fermion" (3)	1836,281044	938,33767 MeVc ⁻²	proton
"boson" (5), $L=18$	1836,281044	938,33767 MeVc ⁻²	proton
"boson" (5), $L=9,767$	2,44726 · 10 ⁵	125,05475 GeVc ⁻²	Higgs boson
"boson" (5), $L=17,9997...$	4,8912219891 · 10 ²²	2,499409289 · 10 ¹⁹ GeVc ⁻²	Plancks mass
"boson" (5), $L=17,9997...$	1836,1676	938,26438 MeVc ⁻²	proton
"boson" (5), $L=18,00619...$	2,4265 · 10 ¹⁷	1,2399 · 10 ¹⁴ GeVc ⁻²	Mass of monopôle?
"boson" (5), $L=18,00619...$	1838,683562	939,5653624 MeVc ⁻²	neutron
"gravitation" (8), $L=18,00619...$	7,14153971 · 10 ⁵	364,932 GeVc ⁻²	4x boson Z ⁰
"gravitation" (8), $L=17,875$	72327,56	36,9593 GeVc ⁻²	?
"gravitation" (8), $L=17,875$	7711,725	3,94 GeVc ⁻²	?
"gravitation" (8), $L=17,75$	4520,47	2,31 GeVc ⁻²	?
"gravitation" (8), $L=17,75$	183195,12	93,61 GeVc ⁻²	≈ boson Z ⁰
"fermion+boson" (12), $L≠0$	0	0	foton
"fermion+boson" (12), $L=9,369$	314595,5584	160,758 GeVc ⁻²	2x boson W [±]
"fermion+gravitation" (13), $L=0$	22579,67	11,5375 GeVc ⁻²	?
"fermion+gravitation" (13), $L=18$	4,507 · 10 ⁶	2,3 TeVc ⁻²	?
"fermion+gravitation" (13), $L=24$	2,2.. to 75009,1	1,133 MeVc ⁻² to 38,3 GeVc ⁻²	dark energie?
"boson+gravitation" (15)	11387	5,819 GeVc ⁻²	dark mass?